

Mechanism Design for Capacity Allocation with Price Competition

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ABSTRACT

Studies on mechanism design mostly focus on a single market where sellers and buyers trade. This paper examines the problem of mechanism design for capacity allocation in two connected markets where a supplier allocates products to a set of retailers and the retailers resale the products to end-users in price competition. We consider the problems of how allocation mechanisms in the upstream market determine the behaviors of markets in the downstream market and how pricing policy in the downstream market influences the properties of allocation mechanisms. We classify an effective range of capacity that influences pricing strategies in the downstream market according to allocated quantities. Within the effective capacity range, we show that the retailers tend to inflate orders under *proportional allocation*, but submit truthful orders under *uniform allocation*. We observe that heterogeneous allocations results in greater total retailer profit which is a unique phenomenon in our model. The results would be applied to the design and analysis of Business-to-Business (B2B) marketplaces and supply chain management.

Categories and Subject Descriptors

K.4.4 [Computers and Society]: Electronic Commerce;
I.2.1 [Artificial Intelligence]: Applications and Expert Systems—*Games*

General Terms

Design, Economics, Management

Keywords

Allocation mechanism design, supply chain management, oligopoly

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1. INTRODUCTION

Mechanism design (MD) has been one of the most promising research topics in the fields of e-business and artificial intelligence in recent years [8, 26, 22, 13]. The main concern of MD is to induce truthful preferences from self-interested agents. Studies on MD mostly focus on a single market where sellers and buyers trade. However, in many situations, traders' preferences are restricted to market situations. If the traders are located in a supply chain, the information about the traders' preferences can then be partially deduced from the analysis of the market situations. Particularly, it is interesting to show how such information could be applied to mechanism design. In this paper we explore the problem in the context of capacity allocation mechanisms.

Consider a supply chain where a supplier sells products to a set of retailers. Suppose all orders from the retailers exceed the capacity of the supplier. To solve unbalance of supply and demand, allocation instead of an adjustment by price is commonly observed in many supply chains. This is particular in the upper stream of supply chains such as component, raw material and natural resource markets. The reasons that allocation is favored in the upstream is changing price are time consuming and costly for the customers. A change of price of raw material involves the product cost controlling of the customers and, in the worst case, the customer must go back to a product design phase. In fact, a long-term price contract is common and a spot price contract is observed for a small amount of trades in such markets. Hence, the price adjustment is not always proper method. When a commodity supplier determines to allocate products, a number of working staffs in a sales and operations department ask customers about the truthful demand. This activity is very common in industry and a typical procedure when the supplier employs *proportional allocation mechanism* which is the most popular mechanism. We show a reason why this procedure is required under the mechanism. Furthermore, we present how to induce truthful orders from the retailers by an allocation mechanism.

There are two reasons for us to choose the subject. First, capacity allocation is one of the most important issues in computing-related applications, such as resource allocation [9, 1, 15], task allocation [17, 23]. Moreover, capacity allocation deals with the problem of scarce resources. A small change of allocation in the upstream market would affect the downstream markets significantly. Therefore the design

of allocation mechanism has to take the whole supply chain into account.

In this paper, we consider a supply chain with two-connected markets where a supplier allocates products to a set of retailers (upstream market) and the retailers resale the products to end-users (downstream market). We assume that the upstream market applies a certain allocation mechanism and the downstream market applies a price competition. Our interests are how the allocation mechanism applied in the upstream market influences the competition among the retailers in the downstream market and how a variation of prices in the downstream market affects the behavior of the allocation mechanism in the upstream market. We restrict our analysis to two most popular and widely used allocation mechanisms in industry, proportional allocation and uniform allocation [7, 6]. We investigate the behavior of these two allocation mechanisms by calculating the equilibria of the retailers' order quantities and the price equilibria.

Our model is differed from a typical single-market model such as [12, 31, 19, 27] with respect to the orders from the retailers. In our model, the truthful orders are not simply determined from a given preference and an employed mechanism. Due to a market competition, we take into account strategic interactions among the retailers for the order determination. In our setting, a comprehension of market rules in the downstream market is important to design allocation mechanisms in the upstream market, because the order quantities of retailer are restricted to the competition in the downstream market. In other words, the order quantity does not only represent private information, but also it reflects the effect of the price competition. We present that the properties of allocation mechanisms are highly related to market rules in the successive market. Cachon and Lariviere [6] have made a contribution to the study of allocation mechanism design in supply chain network based on the assumption that the retailers in the downstream market enjoy local monopolies. With such an assumption, each retailer does not face in a direct competition. Therefore the order quantities of the retailers become purely private information similarly to the single-market model. It implies that no extra information from the downstream market is required to design truth-inducing mechanisms.

Furuhata and Zhang [11] consider the problem by introducing competition into the downstream market. They consider a simple case where the downstream market is in quantity competition. However, in the real-world, price competition is more commonly applied market mechanism and much more complicated in conjunction with allocation mechanism. The complexity comes from the fact that retail price is determined by the market supply, i.e., the supplier's capacity, in quantity competition, while individual sellers do determine their selling prices in price competition. In fact, we observe that the largest retailer can increase its selling price in spite of having unsold products which even leads a greater overall profit (see Theorem 7). This phenomenon diverges not only from Cournot quantity competition but also from Bertrand price competition.

This paper is organized as follows. Section 2 presents our model. In Section 3, we classify effective ranges of the capacity that influence pricing strategies in the downstream market according to allocation quantities. In Section 4, we show how the retailers place orders according to allocation mechanisms and how retailers determine the retail prices.

Then, in Section 5, we focus on how allocation mechanisms affect the total retailer profit. Section 6 briefly concludes this paper.

2. MODEL

We consider a supply chain model with two connected markets: a monopolistic *upstream market* (wholesale market) and an oligopolistic *downstream market* (retail market) as shown in Figure 1. In the upstream market, a supplier sells products to intermediaries, called retailers. When orders from the retailers exceed the capacity size of the supplier, the supplier allocates products according to a selected allocation mechanism. In the downstream market, the retailers resale products to a range of end-users in price competition. We investigate how a competitive model in the downstream market affects properties of the allocation mechanisms in the upstream market and how the allocation mechanisms affect behaviors of the successive downstream market. With such a supply chain model, the capacity allo-

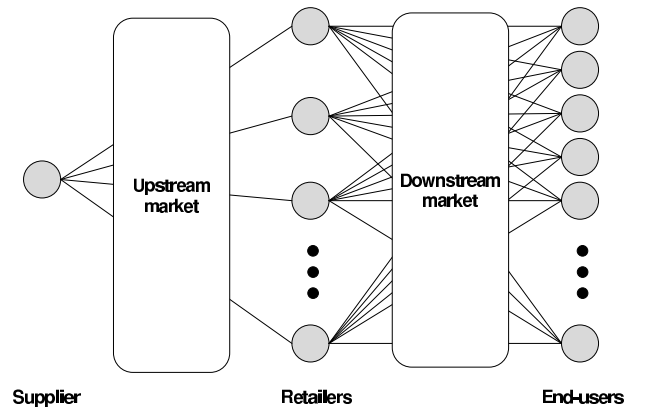


Figure 1: Supply Chain Model

cation problem consists of two stages: order placement and allocation in the upstream market; price setting and resale to end-users in the downstream market.

2.1 Upstream Market

In the first stage, the supplier sets its capacity exogenously denoted by K based on its own demand forecast, i.e. the supplier does not know either a demand function of end-user or how many quantities the retailers will order. Once capacity K is determined, it is not able to change afterwards. Then, the supplier selects an allocation mechanism denoted by g by which the supplier allocates product where the capacity is bound. The supplier notifies the selected allocation mechanism to all retailers. Let $N = \{1, \dots, n\}$ be the set of retailers usually noted i, j or k . Let us denote $j \neq i$ as all retailers except for retailer i . Let m_i be an order quantity of retailer i and let us denote $-i$ as sum of all quantities except for the quantity of retailer i , for instance, $m_{-i} = \sum_{j \neq i} m_j$. Retailer i determines order quantity m_i with respect to market demand, allocation mechanism, and other retailers' orders to maximize its profit denoted by π_i . Let

us denote revenue of retailer i as Π_i . All retailers submit their orders, $m = (m_1, \dots, m_n)$, simultaneously and independently. We assume a wholesale price denoted by w is fixed, same for all retailers and determined exogenously. If the total order quantity exceeds K , the supplier allocates products according to the adopted allocation mechanism. Let $\mathcal{A} = \{a \in \mathbb{R}^n : a \geq 0 \ \& \ \sum_{i=1}^n a_i \leq K\}$, where a vector $a \geq 0$ means for any component a_i of the vector, $a_i \geq 0$. We call each $a \in \mathcal{A}$ a *feasible allocation*.

DEFINITION 1. *An allocation mechanism is a function $g : \mathbb{R}^n \rightarrow \mathcal{A}$ which assigns a feasible allocation to each vector of orders such that for any retailers' order vector m , $g_i(m) \leq m_i$ for each $i = 1, \dots, n$.*

Let $g_i(m^*)$ be the allocation quantity of retailer i under allocation mechanism g with respect to the vector of the equilibrium order quantity m^* . An allocation g is said to be *efficient* if $\sum_{i=1}^n g_i(m) = K$ whenever $\sum_{i=1}^n m_i \geq K$. For the supplier, the efficient allocation is the preferable one since the capacity has been fully used.

The main concerns of mechanism design are efficiency and stability. For a supplier, capacity utilization is a key performance index to evaluate its performance. If a mechanism makes the capacity utilization increase, it is a preferable mechanism for the supplier. A typical mechanism design criterion in capacity allocation is *individually responsive* (IR) which contributes to increase the capacity utilization.

DEFINITION 2. *An allocation mechanism g is said to be IR if for any i ,*

$m'_i > m_i$ and $g_i(m) < K$ imply $g_i(m'_i, m_{j \neq i}) > g_i(m_i, m_{j \neq i})$ where m is a vector of retailers' orders, m_i is the i 's component of m , $m_{j \neq i}$ is a vector of the other retailers' orders, and m'_i is a variation of m_i .

Under IR mechanisms, a retailer receives more allocations if it orders more. Consequently, retailers frequently place more orders than they actually need. On the other hand, the inflated orders prevent a right evaluation of the capacity investment. When demand is unstable, the supplier often fails to make decision on capacity planning due to lack of truthful order information. Hence, a mechanism inducing truthful order information from retailers is desirable mechanism design criteria. This criterion is formally represented as follows:

DEFINITION 3. *An allocation mechanism g is said to be incentive compatible (IC) or truth-inducing if all retailers placing orders truthfully at their optimal sales quantities is a Nash equilibrium of g , formally for all i :*

$$\pi_i(g_i(m^*), g_{j \neq i}(m^*)) \geq \pi_i(g_i(m_i, m_{j \neq i}^*), g_{j \neq i}(m_i, m_{j \neq i}^*)).$$

2.2 Downstream Market

In the second stage, the retailers are in the price competition in the downstream market. Let $D(p)$ be the demand of the end-users at price p and $P(q)$ be its inverse function where q stands for the total supply quantity. We assume that the function $P(q)$ is strictly positive on some bounded interval $(0, \hat{q})$, on which it is twice-continuously differentiable, strictly decreasing and concave. For $q \geq \hat{q}$, we

simply assume $P(q) = 0$. In order to focus on interesting cases, we assume retailers only have purchase costs w where $w < P(K)$. Let a_i be the allocated quantity for retailer i which has been determined in the first stage. Once retailers are allocated by the supplier, the retailers determine prices p_i simultaneously and independently. Like Levitan and Shubik [18] and Kreps and Scheinkman [16], we assume *surplus maximizing rule* where the end-users choose from the lowest price-offering retailers.

Here, we introduce following notations based on Cournot quantity competition in order to describe a price competition. Let q_{-i} stands for $\sum_{j \neq i \in N} q_j$. We define the best response function for retailer i at cost w in Cournot quantity competition as:

$$r_w(q_{-i}) = \arg \max_{q_i} \{q_i P(q_i + q_{-i}) - wq_i\}. \quad (1)$$

We assume that $q_i P(q_i + q_{-i}) - wq_i$ is concave in q_i for all q_{-i} . Based on this assumption, $r_w(q_{-i})$ is a unique solution of $P(q_i + q_{-i}) + q_i \frac{dP(q_i + q_{-i})}{dq_i} - w = 0$ and satisfies

$$-1 < \frac{\partial r_w(q_{-i})}{\partial q_{-i}} < 0. \quad (2)$$

Hence, $r_w(q_{-i}) + q_{-i}$ is increasing in q_{-i} . Let us denote Cournot equilibrium at cost w as q^{cw} and the total Cournot quantity as $q^{Cw} = \sum_{i=1}^N q^{cw}$. Note that $r_w(q_{-i}^{cw}) = q^{cw}$. At the special case where cost is zero, we denote q^c as the Cournot equilibrium at zero cost and q^C as the total Cournot quantity.

From now on, we show how retailers set the price. Let us consider from the case where the supplier allocates the product exclusively. Let i be the exclusively allocated retailer enjoying the benefits of the monopoly price p^M . Since $qP(q)$ is concave in q , where $q \in (0, \hat{q})$, we have

$$q^M = \operatorname{argmax}_q qP(q). \quad (3)$$

Therefore, the selling quantity is $x_i = \min\{q^M, a_i\}$. Hence retailer i sets the monopoly price $p^M = P(x_i)$ to maximize its profit $\pi_i = x_i P(x_i) - wa_i$.

Now let us consider the case where the capacity is allocated to several different retailers. The remaining part of the problem is how the retailers determine retail prices according to the allocation. This pricing problem is similar to a model proposed by Francesco [10] where several manufacturers compete in price in an oligopolistic market with capacity pre-commitments. When the pre-commitment of the capacity exceeds the best response quantity, the manufacturer considers two options which are: (i) selling all products at a lower price or (ii) selling a limited quantity at a higher price. Francesco shows how the manufacturers choose the options. Based on the option, Francesco shows price equilibria that are dependent on the pre-committed capacity sizes. Notice that the manufacturers in Francesco correspond to the retailers in our model and the capacity pre-commitments correspond to the allocation quantities. Before we describe the relationship between allocation quantity and price equilibria, we show the relationship between allocation and best response (the lemma is based on Boccard and Wauthy [4, 5]).

LEMMA 1. *Given an efficient allocation mechanism and*

let a be the allocated quantities. Suppose $a_i > a_j$. If $a_i \leq r(a_{-i})$ then $a_j < r(a_{-j})$.

PROOF. According to suppositions, we have $a_{-i} < a_{-j}$. According to Equation (2), we have $r(a_{-i}) < r(a_{-j})$. Hence we have $a_j < a_i \leq r(a_{-i}) < r(a_{-j})$. \square

According to lemma 1, if allocation a_i is less than or equal to its best response, the smaller allocation quantities satisfy the same relationships. Furthermore, it implies, if the largest allocation a_l satisfies $a_l \leq r(a_{-l})$, then all the other allocations have the same characteristic.

In [10], Francesco investigates price equilibria in both pure strategy and mixed strategy. Now let us denote, \bar{p} and \underline{p} as an upper and a lower bound of the price equilibrium in the mixed strategy. The following lemma characterizes the price equilibria that are a straightforward conversion from the capacity sizes of the manufacturers in the model of Francesco [10] to the allocated quantities of retailers in terms of our model,

LEMMA 2. [10] Given an allocation mechanism g , and order m , let $a_i = g_i(m)$. Let the largest allocated retailer $l = \operatorname{argmax}_{i \in N} a_i$. Then

1. if for all i , $a_i \leq r(a_{-i})$, $P(a_i + a_{-i})$ is a unique equilibrium.
2. if $a_l > r(a_{-l})$ and $D(0) > a_{-l}$, then

$$\bar{p}_l = P(r(a_{-l}) + a_{-l}),$$

$$\underline{p}_i = \frac{P(r(a_{-l}) + a_{-l})r(a_{-i})}{a_i} \text{ for all } i, \text{ and}$$

$$\Pi_l = r(a_{-l})P(r(a_{-l}) + a_{-l}),$$
3. if $D(0) \leq a_{-l}$, $p^* = 0$ is the unique price equilibrium.

PROOF. If we view i and a_i as a certain manufacturer and its respective capacity choice in [10], the pricing problem in the downstream market can be seen as the same problem as Francesco [10]. See proofs of Propositions 1 and 2 in [10]. \square

According to Lemma 2, Francesco shows that there are three types of price equilibria according to the patterns of allocations in our context. First, if an allocation quantity for each retailer is less than or equals to its best response quantity, the retailers set the price $P(a_i + a_{-i})$. Second, if the allocation quantity for the largest allocated retailer is greater than its best response quantity and the sum of the allocations for the rest of retailers does not exceed the maximum demand, the retailers set the price between \underline{p} and \bar{p} in mixed strategy. Third, if the sum of the allocations for the retailers except for the largest allocated retailer is greater or equals to the maximum demand, the retailers set price zero. Once we have the allocation quantities for the retailers, according to Lemma 2, we know how the retailers choose pricing strategies and the price equilibria in the downstream market.

Here, let us describe the difference between Bertrand price competition and our price competition having capacity allocation as constraints for the sales. In Bertrand competition, the sellers do not have any upper bound for the selling quantities. Hence, the equilibrium price becomes zero. For example, if there is one seller sets the price lower than the others, the lowest price offering seller dominates the whole market. This is an incentive for the sellers to set lower prices. Contrary, the retailers have upper bounds (allocations) for the

selling quantities in our model. Therefore, there are some residual demands for the retailers offering higher price under certain circumstances. Hence, the price equilibria are not always zero in our model as shown in Lemma 2.

Allocation quantity is one of the most influential factors to determine selling prices. We address an issue: how many allocation quantities are desirable in an oligopolistic price competition with constraints? We follow the result of Kreps and Scheinkman [16] to deal with the issue. The model of Kreps and Scheinkman is a duopoly version of Francesco's model. Kreps and Scheinkman show that the equilibrium of the capacity pre-commitment is the Cournot quantity q^{cw} . They assume that the manufacturers are able to determine their amounts of capacity pre-commitments independently. This assumption means that we do not have allocation constraints in our model. It implies that we are able to view the Cournot quantity as a truthful demand in our model.

In the next section, we extend Lemma 2 in order to establish a link between capacity sizes and price equilibria which is useful to know the effective range of capacity for pricing strategies.

3. EFFECTS OF CAPACITY ALLOCATION TO THE DOWNSTREAM MARKET

Our interests are how capacity allocation in the upstream market influences the market behaviors in the downstream market and how the market rules influence the properties of allocation mechanisms. Since it is not simple to find an order equilibrium and a price equilibrium under certain allocation mechanisms, this section aims to shrink a problem. In this section, we find a price equilibrium according to the capacity size. In other words, we aim to comprehend how the retailers determine their prices under different capacity settings. In this section, we assume that the order quantity is given and the total order quantity exceeds the capacity, i.e. g is efficient. Since the order quantity is given, we treat the purchase cost as sunk cost.

Prior to investigation of the relationship between the supplier's capacity and the retailers' pricing strategies, we would like to emphasize some typical economical indicators which have been shown in Section 2, such as q^M : the monopoly quantity of the downstream market, q^C : the total Cournot quantity in the oligopoly market, \hat{q} : the maximum demand of the downstream market, and $P(K)$: the market price corresponding to the quantity of K .

We divide the capacity ranges in four ranges for the analysis.

- Strictly scarce capacity ($K \leq q^M$): the supplier's capacity K is less than the monopoly quantity q^M of the downstream market.
- Relatively scarce capacity ($q^M < K \leq q^C$): the capacity is greater than the monopoly quantity and less than or equal to the total Cournot quantity q^C .
- Enough capacity ($q^C < K < \hat{q}$): the capacity is greater than the total Cournot quantity and less than the maximum demand of the downstream market.
- Excessive capacity ($\hat{q} \leq K$): the capacity is greater than the maximum demand.

The following theorem deals with the first situation where the capacity of the supplier is very limited (i.e. its capacity is less than the monopoly quantity).

THEOREM 1. *Given an allocation mechanism g . Suppose that $K \leq q^M$ and g is efficient. If g is feasible, then $P(K)$ is an equilibrium, i.e., $p^* = P(K)$*

PROOF. According to Lemma 2, it is sufficient to show that for any i , $g_i(m) \leq r(g_{-i}(m))$. Since g is efficient and $K \leq q^M$, we have $g_i(m) + g_{-i}(m) = K \leq q^M = \underset{q}{\operatorname{argmax}} qP(q)$, with respect to Equation (3).

Case 1: If $r(g_{-i}(m)) + g_{-i}(m) \geq q^M$, we have

$$g_i(m) + g_{-i}(m) \leq r(g_{-i}(m)) + g_{-i}(m).$$

It follows that $g_i(m) \leq r(g_{-i}(m))$ as desired.

Case 2: Assume that $r(g_{-i}(m)) + g_{-i}(m) < q^M$. According to the definition of Cournot best response function, we have $r(g_{-i}(m)) = \underset{q_i}{\operatorname{argmax}} q_i P(q_i + g_{-i}(m)) = \underset{q_i}{\operatorname{argmax}} ((q_i + g_{-i}(m))P(q_i + g_{-i}(m)) - g_{-i}(m)P(q_i + g_{-i}(m)))$. Let $y = q_i + g_{-i}(m)$ and $y^* = \underset{y}{\operatorname{argmax}} (yP(y) - g_{-i}(m)P(y))$; we have:

$$\begin{aligned} r(g_{-i}(m)) &= \underset{y}{\operatorname{argmax}} (yP(y) - g_{-i}(m)P(y)) - g_{-i}(m) \\ &= y^* - g_{-i}(m) \end{aligned} \quad (4)$$

According to the assumption of Case 2, Equation (4) implies $y^* < q^M$. It follows that $P(y^*) > P(q^M)$ because P is strictly decreasing in quantity. It turns out that

$$-g_{-i}(m)P(y^*) < -g_{-i}(m)P(q^M)$$

Notice that the case $g_{-i}(m) \leq 0$ is ruled out since $y^* < q^M$. On the other hand, $y^*P(y^*) \leq q^M P(q^M)$ because $q^M = \underset{q}{\operatorname{argmax}} qP(q)$. Therefore, we have,

$$y^*P(y^*) - g_{-i}(m)P(y^*) < q^M P(q^M) - g_{-i}(m)P(q^M)$$

This contradicts the definition of y^* . That is $r(g_{-i}(m)) + g_{-i}(m) \geq q^M$ and we have $g_i(m) \leq r(g_{-i}(m))$. \square

This theorem shows that the equilibrium price is $P(K)$, when the capacity size of the supplier is less than the market monopoly quantity q^M and allocation mechanism is efficient. In this case, no retailers can make greater profit by charging higher price than $P(K)$, which is the monopoly price when $K \leq q^M$. Therefore, the allocation mechanism does not affect the market price within the capacity range. In other words, in this context, the supplier has no interests to choose a specific allocation mechanism as long as the mechanism is efficient.

Next, we consider the case where the capacity is relatively scarce that is $q^C < K \leq q^C$. We have the following result:

THEOREM 2. *Suppose $q^M < K \leq q^C$. Under any efficient allocation mechanism g , price equilibria are,*

1. $p^* = P(K)$, if for all i $g_i(m) \leq r(g_{-i}(m))$;
2. $p^* > P(K)$, if there exists i such that $g_i(m) > r(g_{-i}(m))$.

PROOF. Since g is efficient, we have $\frac{K}{n} \leq g_l(m) \leq K$. It implies $\frac{n-1}{n}K \geq g_{-l}(m) \geq 0$. Since $K < q^C$, we have $\frac{n-1}{n}K \leq q_{-i}^c$. According to Equation (2), we obtain $r(q_{-i}^c) \leq$

$r(\frac{n-1}{n}K) \leq r(g_{-l}(m)) \leq r(0) = q^M$. Therefore, we have either $g_l(m) \leq r(g_{-l}(m))$ or $g_l(m) > r(g_{-l}(m))$. In the former case, according to Lemma 1 and Lemma 2 case 1, we have $p^* = P(K)$. In the later case, according to Lemma 2 case 2, we have $p^* > P(K)$. \square

According to Theorem 2, there are two types of pricing strategies for the retailers, when the capacity is relatively scarce. If allocation quantities for each retailer do not exceed the best response quantities, the market price is stable and the equilibrium price reaches the price at the capacity size, which is the statement 1. Otherwise, the market price becomes unstable and higher than $P(K)$, which is the statement 2. This is a very interesting phenomenon. If there is a retailer enjoying privilege from unbalance of allocation, the equilibrium price in the downstream market is higher than the balanced case. The following two extreme cases help to understand the difference. If the supplier allocates the capacity exclusively to a retailer which is one case of the statement 2, the retailer sets the resale price as $P(q^M) > P(K)$ to maximize its profit, even if the retailer is not able to sell all the products. On the other hand, if the supplier allocates the capacity to all retailers equally, which is one case of the statement 1, the mechanism of the price competition works properly and the retailers are not able to increase their profits by charging higher prices than $P(K)$.

We turn to investigate a case where $q^C < K < \hat{q}$ in the following theorem,

THEOREM 3. *Suppose $q^C < K < \hat{q}$. For any efficient allocation mechanism g , $p^* > P(K)$.*

PROOF. Since $K < \hat{q}$, for any g , we have $g_l(m) \leq K < \hat{q}$. Suppose, for all i , we have $g_i(m) \leq r(g_{-i}(m))$. According to the definition of Cournot best response function, we obtain $g_i(m) \leq q_i^c$. It implies $\sum_{i=1}^n g_i(m) \leq q^C$ which is a contradiction of the supposition of an efficient allocation $\sum_{i=1}^n g_i(m) = K$. Hence, for retailer l , we have $g_l(m) > r(g_{-l}(m))$. According to Lemma 1 and Lemma 2 case 2, we have $p^* > P(K)$. \square

According to Theorem 3, the retailers set the resale price greater than $P(K)$ if the capacity is in $q^C < K < \hat{q}$. In this case, even if the allocation is equal to all retailers, the allocation quantity is greater than the best response quantity. Hence, the retailer sets the price higher than $P(K)$ similarly to the Statement 2 in Theorem 2.

Finally, we show the case $K \geq \hat{q}$.

THEOREM 4. *Suppose $K \geq \hat{q}$. For any efficient allocation mechanism,*

1. $p^* = 0$, if $g_{-l}(m) \geq \hat{q}$.
2. $p^* > 0$, otherwise.

PROOF. Since $K \geq \hat{q}$ and g is efficient, g satisfies either $g_{-l}(m) \geq \hat{q}$ or $0 \leq g_{-l}(m) < \hat{q}$. The first case is the condition of Lemma 2 case 3. Hence, we have $p^* = 0$. In the later case, since $\frac{K}{n} > q^c$, we obtain $p^* > P(K)$ as same as Theorem 3. \square

According to Theorem 4, if $\hat{q} \leq K$ and if the sum of the all retailers' allocations excepts for the largest allocation exceeds the maximum demand, the equilibrium price is zero. This is similar to the result of Bertrand price competition. On the other hand, if the sum of the all retailers' allocations excepts for the largest allocation that does not exceed the maximum demand, the retailers are able to earn some profits by setting the price higher than zero, but the profits are very limited.

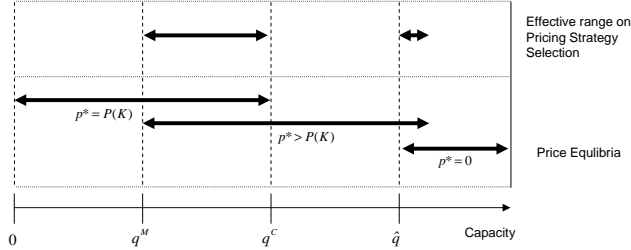


Figure 2: Capacity and Corresponding Pricing

Now, let us summarize the results of this section. We have classified how capacity allocation in the upstream market affects the pricing strategy and the price equilibria in the downstream market as shown in Figure 2. The effective range of the capacity for the pricing strategy selection is $q^M < K \leq q^C$ and a special case $\hat{q} < K$ and $g_{-l}(m) < \hat{q}$. A crucial range of capacity that affects the pricing strategy selection is $q^M < K \leq q^C$. Within the capacity range, the allocation mechanism selection is remarkably sensitive to the downstream market. If $K \leq q^M$, the equilibrium price is the monopoly price. If $q^M < K \leq q^C$, allocations gives a significant impact on the price equilibria. If $q^C < K < \hat{q}$, the price equilibria are sensitive to allocations. In the next section, we relax the assumptions that are the given order quantities and the total order quantity exceeds the capacity.

4. EFFECTS OF ALLOCATION MECHANISM IN SUPPLY CHAIN

In the previous section, we have classified how capacity allocation affects the pricing strategies and the price equilibria in the downstream market corresponding to the capacity size. In this section, we consider how the retailers set order quantities based on allocation mechanisms and how the retailers determine selling prices according to allocated quantities in the supply chain. Particularly, we focus on two popular allocation mechanisms in industry, uniform allocation and proportional allocation. Notice that the truthful order quantity is q_i^{cw} as shown in Section 2.

A main goal of this model with respect to mechanism design aspect is to obtain a truth-inducing mechanism. We investigate market behaviors under uniform allocation presented in [28], which is a truth-inducing mechanism in [6] and [11]. Under uniform allocation, the retailers are indexed in ascending order of their order quantity, i.e., $m_1 \leq m_2 \leq \dots \leq m_n$. Let

$$\lambda = \max \left\{ i : K - nm_1 - \sum_{j=2}^i (n-l)(m_j - m_{j-1}) > 0 \right\} \text{ and}$$

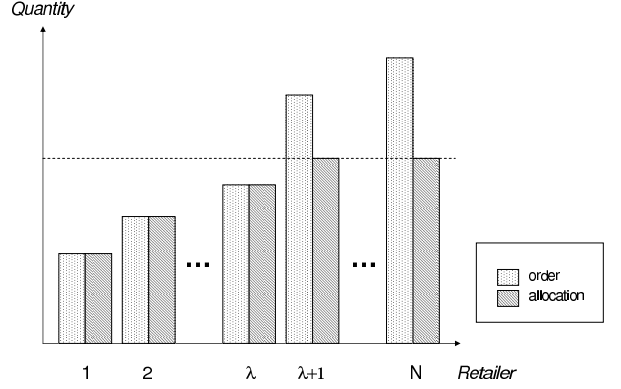


Figure 3: Uniform Allocation Mechanism

uniform allocation is,

$$g_i(m) = \begin{cases} K/n, & \text{if } nm_1 > K, \\ m_i, & \text{if } i \leq \lambda, \\ m_\lambda + \left(K - (n - \lambda + 1)m_\lambda - \sum_{j=1}^{\lambda-1} m_j \right) / (n - \lambda), & \text{otherwise.} \end{cases}$$

Under uniform allocation mechanism, the retailers with orders less than a threshold m_λ receive the same quantities as respective orders, and the rest of retailers receive m_λ and the rest of capacity divided by the number of retailers ordered greater than m_λ . The threshold of m_λ is led by the following procedure. If $m_1 \times n$ is greater than the capacity, all retailers receive $\frac{K}{n}$, otherwise, there is a threshold m_λ where λ is greater than or equal to 1. In case of $m_1 \times n \leq K$, the supplier counts up the number of retailers, while $\sum_{j=1}^i m_j + m_j \times (n - i) \leq K$ (the sum of i -th smallest orders and the quantity of the i -th order times the number of the rest of retailers is less than the capacity size of the supplier). If the sum of orders is greater than or equal to the capacity, the allocation quantity is equal to the capacity, which is the area below the horizontal dashed line in Figure 3.

We have the following order quantity equilibria and the price equilibria under uniform allocation.

THEOREM 5. *Under uniform allocation, there is a unique equilibrium order quantity $m_i^* = q_i^{cw}$ in the upstream market, which induces price equilibria $p_i^* = \max\{P(K), P(q^{cw})\}$ in the downstream market.*

PROOF. If $m_i^* = q_i^{cw}$ for all i , according to the definition of uniform allocation, we have $g_i(m^*) = \min\{q_i^{cw}, K/n\} \leq q^{cw}$. Since $g_i(m^*) \leq r_w(g_{-i}(m^*)) < r(g_{-i}(m^*))$, the profit of retailer i is

$$\pi_i = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*). \quad (5)$$

If $m_i' > m_i^*$ and $m_{j \neq i}^* = q^{cw}$, we have $g_i(m') = g_{j \neq i}(m') = K/n$, which is the same allocation quantity to the case at m^* .

Hence, by increasing order m'_i , retailer i cannot increase its profit. If $m'_i < m_i^*$, we have $g_i(m') \leq \frac{K}{n} \leq g_{j \neq i}(m') \leq q^{cw}$. We check whether this case fits to the condition of case 2 of Lemma 2. According to Equation (2), we have $r(g_{-i}(m')) \leq r(g_{-j}(m'))$. Since $g_{j \neq i}(m') \leq q_j^{cw} = r_w(q_{-j}^{cw}) < r(q_{-j}^{cw}) \leq r(g_{-i}(m')) \leq r(g_{-j}(m'))$, this case does not satisfy the condition of case 2 of Lemma 2. Hence, we only consider the case of the pure strategy. The profit of retailer i is same as Equation (5). Recall that Equation (5) is concave in $g_i(m)$ and maximized at $r_w(g_{-i}(m))$. Since $g_i(m') \leq r_w(q_{-i}^{cw})$, by decreasing m'_i , π_i is not increased. Therefore, we have an equilibrium order quantity $m_i^* = q_i^{cw}$, an allocation quantity at $g_i(m^*) = \min\{K/n, q_i^{cw}\}$, and an equilibrium price $p_i^* = \max\{P(K), P(q^{Cw})\}$. \square

According to Theorem 5, all retailers place truthful order quantities q^{cw} under uniform allocation, since no retailers are able to increase their profits by increasing orders or decreasing orders from the truthful order quantity. An interesting property of uniform allocation is its robustness of allocation at the truthful order quantity. Even if competitors increase their order quantities, the allocation quantity for the retailer that submits the truthful order is not decreased. At the equilibrium order quantity m^* , we have allocation quantities $g_i(m^*) = \min\{K/n, q_i^{cw}\}$. Since the allocation quantity for each retailer does not exceed the best response quantities, we have the price equilibria $p_i^* = \max\{P(K), P(q^{Cw})\}$ under uniform allocation.

Uniform allocation satisfies the truth-inducing property. This is a very important criterion to choose an allocation mechanism for mechanism designers. However, in most cases, the supplier distributes products for several different markets. Hence, proportions to fulfill the demands by allocations are different under uniform allocation. An undesirable point for the supplier is that the larger market is allocated less proportion compared to the smaller market under uniform allocation.

In industry, the most commonly used allocation is proportional allocation, which is a representative IR allocation mechanism. An allocation mechanism g is proportional allocation if

$$g_i(m) = \min \left\{ m_i, K m_i / \sum_{j=1}^N m_j \right\}. \quad (6)$$

In other words, whenever capacity binds, allocated quantity to each retailer is the same fraction of its order under the proportional allocation. Similarly to Cachon and Lariviere [6], we show that the retailers inflate orders and there is no equilibrium under proportional allocation in our model.

THEOREM 6. *There does not exist an order equilibrium m^* under proportional allocation, if $K \leq q^{Cw}$.*

PROOF. Suppose there exists symmetric m^* . Let us denote $\pi_i(g(m)) = \pi_i(g_i(m), g_{j \neq i}(m))$. Since $K \leq q^{Cw}$, we have $g_i(m^*) \leq q^{cw}$ and $g_{-i}(m^*) \leq q_{-i}^{cw}$. It turns out $r(g_{-i}(m^*)) \geq r(q_{-i}^{cw})$. Since $g_i(m^*) \leq q_i^{cw} = r_w(q_{-i}^{cw}) < r(q_{-i}^{cw}) \leq r(g_{-i}(m^*))$, the profit of retailer i is $\pi_i(g(m^*)) = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*)$, according to Lemma 2 case 1. Let $m' = (m'_i, m_{j \neq i}^*)$. Equation (6) implies $g_i(m') > g_i(m^*)$ where $m'_i > m_i^*$.

If $\sum_{i \in N} m_i^* < K$, we have $g_l(m')$ such that $g_l(m') < q_l^{cw}$. Equation (6) implies $g_{-l}(m') \leq g_{-l}(m^*)$ and $r(g_{-l}(m')) \geq$

$r(g_{-l}(m^*))$. It turns out that $g_l(m') < q_l^{cw} = r_w(q_{-l}^{cw}) < r(g_{-l}(m^*)) \leq r(g_{-l}(m'))$. According to Lemma 2 case 1, we have $\pi_l(g(m')) = P(g_l(m') + g_{-l}(m'))g_l(m') - wg_l(m')$. According to the concavity of the profit function and $g_i(m^*) < g_l(m') \leq r(g_{-l}(m'))$, we have $\pi_l(g(m')) > \pi_l(g(m^*))$. Hence, symmetric m^* does not exist in the case of $\sum_{i \in N} m_i^* < K$.

If $\sum_{i \in N} m_i^* \geq K$, Equation (6) implies $g_l(m') - g_l(m^*) = -(g_{-l}(m') - g_{-l}(m^*))$ where $m'_i > m_i^*$. It follows $g_l(m') < r(g_{-l}(m'))$ and $\pi_l(g(m')) = P(g_l(m') + g_{-l}(m'))g_l(m') - wg_l(m')$. The difference between $\pi_l(g(m'))$ and $\pi_l(g(m^*))$ is $(g_l(m') - g_l(m^*))(P(K) - w)$. According to the assumption of w , we have $\pi_l(g(m')) > \pi_l(g(m^*))$. Hence, symmetric m^* does not exist in the case of $\sum_{i \in N} m_i^* \geq K$.

Suppose there exists asymmetric m^* such that $g_l(m^*) < r(g_{-l}(m^*))$. The supposition is contradicted similarly to the case of symmetric m^* and $\sum_{i \in N} m_i^* < K$.

Suppose there exists asymmetric m^* such that $g_l(m^*) > r(g_{-l}(m^*))$. The profit of retailer l is

$$\pi_l(g(m^*)) = P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*),$$

according to Lemma 2 case 2. Similarly, the profit of retailer i where $m_i^* < m_i^*$ is

$$\pi_i(g(m^*)) = \frac{P(r(g_{-i}(m^*)) + g_{-i}(m^*))r(g_{-i}(m^*))}{g_i(m^*)} g_i(m^*) - wg_i(m^*).$$

The difference of profits between retailer l and i is

$$\pi_l(g(m^*)) - \pi_i(g(m^*)) = \frac{g_l(m^*) - g_i(m^*)}{g_i(m^*)} (P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*)).$$

At the equilibrium, $\pi_l(g(m^*))$ must be positive. Notice that $\pi_l(g(m^*)) = P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*)$. Hence, we have $\pi_l(g(m^*)) > \pi_i(g(m^*))$. It follows that asymmetric m^* such that $g_l(m^*) > r(g_{-l}(m^*))$ does not exist. \square

According to Theorem 6, there does not exist an equilibrium in order quantity under proportional allocation where the retailers tend to increase their order quantities more than they need. Under proportional allocation, each retailer is able to decrease the competitors' allocations by increasing its order that makes greater profits for each retailer. Thus, proportional allocation is not robust and the supplier receives more order quantities than actual needs.

One way to obtain an order equilibrium under proportional allocation is to assume a maximum order quantity denoted by \tilde{m} . In reality, we frequently encounter the case where there exists a maximum order quantity, which is determined by either the supplier side or the retailer side. In this case, according to Theorem 6, we are easily able to obtain the equilibrium order quantity $m_i^* = \tilde{m}_i$. The interesting

case of the capacity range is $K \leq \sum_{i=1}^n \tilde{m}_i$. If the maximum order quantity is symmetric, we have allocation $g_i(m^*) = \frac{K}{n}$ and the equilibrium price $p^* = P(K)$, which is the same price as the quantity competition shown in [11]. If it is asymmetric, we obtain allocation $g_i(m^*) = K m_i^* / \sum_{i=1}^n m_i^*$. The equilibrium price is either $p^* = P(K)$ or $p^* > P(K)$,

which is dependent on a relationship between the largest allocation $g_l(m^*)$ and its best response $r(g_{-l}(m^*))$. If the maximum order quantities are heterogeneous and the allocation for the largest allocated retailer exceeds its best response quantity, the retailers set their resale price greater than $P(K)$ which is a unique phenomenon in this model compared to the quantity competition in [11].

5. HETEROGENEOUS ALLOCATIONS

In industry, the allocation quantities are not always equal. For example, under proportional allocation with maximum order quantity shown in the previous section, it is commonly observed that some prioritized customers have the greater maximum order quantities. As we have seen in Theorem 2, the difference of allocation quantities to the retailers affects the pricing strategies and the largest allocated retailer increases the selling price to enjoy the benefit of the privileged allocation. This is a unique phenomenon in our model. In this section, we further investigate how heterogeneous allocations affect the total retailer profit.

In the exclusive distribution model in [6], the most important design criteria is Pareto optimality. Here, Pareto optimality means that under allocation mechanism that satisfies Pareto optimal criteria the total retailer profit is maximized, if all retailers submit orders truthfully. However, in our model, to allocate all the products to a single retailer is a way to maximize the total retailer profit. Therefore, Pareto optimality is not a significant criterion in our model. Meanwhile, it does not mean the total retailer profit is not important. The supplier frequently encounters a situation where the supplier needs to give some privileges to some retailers on allocation. We investigate how heterogeneity of allocation make an impact for the total retailer profit.

We focus on the interesting capacity range $q^M < K \leq q^C$ as shown in Section 3. Let us call g^P as a strong heterogeneous allocation which satisfies the condition of case 1 in Lemma 2 and let us call g^M as a non strong heterogeneous allocation for the condition of case 2. According to these conditions, mechanisms g^M reflects the case where some retailers have been privileged. Notice that a symmetric allocation is included in g^P , since $K \leq q^C$.

Let us remind that strong heterogeneous allocation corresponds to statement 2 in Theorem 2 and non strong heterogeneous allocation corresponds to statement 1 in Theorem 2. Within the capacity range $q^M < K \leq q^C$, the retailers have two types of pricing strategies and price equilibria. It is certain that the largest allocated retailer earns greater profit under strong heterogeneous allocation than the profit under non strong heterogeneous allocation. However, we have one question whether the higher equilibrium price under non strong heterogeneous allocation results in the total retailer profit greater. The following theorem corresponds to this question.

THEOREM 7. *Suppose that $q^M < K \leq q^C$. For any efficient $g^P(m)$ and $g^M(m)$, $\sum_{i \in N} \pi_i(g^M(m)) > \sum_{i \in N} \pi_i(g^P(m))$.*

PROOF. Let $\pi_i(g(m)) = \pi_i(g_i(m), g_{j \neq i}(m))$. First we show the total retailers' profit under g^P . According to the

case 1 of Lemma 2, the total retailers' profit is

$$\begin{aligned} \sum_{i \in N} \pi_i(g^P(m)) &= \sum_{i \in N} \left(P(K)g_i^P(m) - wg_i^P(m) \right) \\ &= P(K)K - wK. \end{aligned} \quad (7)$$

Now we show the case of g^M . If $g_l^M(m) \neq K$, according to the case 2 of Lemma 2, the total retailers' profit is

$$\sum_{i \in n} \pi_i(g^M(m)) = \sum_{i \in n} \left(\underline{p}_i g_i^M(m) - wg_i^M(m) \right).$$

We have $\underline{p}^* > P(K)$ under g^M according to Theorem 2. Hence, we have

$$\sum_{i \in n} \pi_i(g^M(m)) = \underline{p}K - wK > P(K)K - wK. \quad (8)$$

According to Equation (7) and (8), we obtain

$$\sum_{i \in n} \pi_i(g^M(m)) > \sum_{i \in N} \pi_i(g^P(m)).$$

If $g_l^M(m) = K$, retailer l is a monopolist. Hence, we have $\pi_l > \sum_{i \in n} \pi_i(g^P(m))$ according to Equation (7) and $K > q^M$. \square

According to Theorem 7, under strong heterogeneous allocation g^M , the total retailer profit is greater than the one under non strong heterogeneous allocation g^P , when the capacity of the supplier is relatively scarce. Since there is no difference with respect to the total cost of all retailers between g^M and g^P , we focus on the revenue. The total revenue is $P(K)K$ under g^P . Meanwhile, the total revenue is $\underline{p}K$ under g^M . At first glance, it seems inconsistent, because the market demand cannot be K if the retail price is $\underline{p} > P(K)$ under g^M . However, $\underline{p}K$ consists of the revenues of all retailers and they do not set price \underline{p} at once. In fact, the total selling quantity under g^M is less than K , since the selling quantity of retailer i at price \underline{p} is less than g_i^M . Notice that the profit of the mixed strategy is equal at any price between \underline{p} and \bar{p} . Therefore, it implies that even if the less prioritized retailers under g^M decrease their profits compared to the ones under g^P , the increasing amount of the profits of the prioritized retailers exceeds the decreasing amount of the less prioritized retailers.

The phenomenon that the heterogeneity of allocation affects the total retailer profit shown above is not observed in the quantity competition model in [11]. In quantity competition, the price is determined by the total supply quantity. Hence, the individual supply quantities do not affect the market price. Hence, the heterogeneity of allocation is not a significant point to consider the total retailer profit in Furuhashi and Zhang's model [11].

6. CONCLUSION AND RELATED WORK

In this paper we studied capacity allocation problems in a supply chain where a supplier allocates capacity to a set of retailers and the retailers compete in price competition in a same market. We showed how the capacity allocations in the upstream market affect the pricing strategy selection and the price equilibria in the downstream market. According to the classification of the effective range of the capacity that affects the market behaviors in the downstream market, we

are able to know the crucial capacity ranges to choose the allocation mechanisms.

With the equilibrium analysis on purchasing and pricing, we investigated the effects of allocation mechanisms in supply chain, especially for two popular allocation mechanisms, uniform allocation and proportional allocation. We found that the equilibrium order quantity would not always be Cournot quantity in our model. This is a significant difference from the result of the Kreps and Scheinkman's model. The difference is observed in a situation where the retailers are able to decrease their competitors' allocation quantities under certain allocation mechanisms such as proportional allocation. Under proportional allocation, the retailers inflate orders to be allocated more than competitors. Contrary, we showed that uniform allocation induces truthful order quantity from the retailers under which we have the same equilibrium price as Furuhata and Zhang [11].

We observed a unique phenomenon in our model that is heterogeneity of allocations results in higher market price if capacity is relatively scarce. Furthermore, the total retailer profit is increased in this case, even though some retailers decrease their profits. In quantity competition, the total market supply determines the market price, thus the price is not affected by heterogeneity of allocations. On the other hand, each retailer determines the selling price in price competition with allocation constraints. Therefore, the privileged retailers are able to take an opportunity to set higher price for the residual demand, because less prioritized retailers are not able to fulfill all the demand. In our model, the total retailer profit is maximized, where the supplier exclusively allocates to a single retailer. It means that one important criteria, Pareto optimality, in Cachon and Lariviere's model [6] is not a significant criteria in our model. Hence, truth-inducing property enhances the significance of mechanism design criteria.

In this model, choosing an allocation is a trade-off between the efficiency goal and the stability goal. IR allocation leads an order inflation that contributes for a higher utilization of the capacity and a greater profit for the supplier. However, the supplier lacks of the accurate demand information. This is a serious problem, since the accurate demand is a fundamental input for all business planning for the supplier. The retailers encounter uncertain allocations and unstable prices under IR allocations. However, there is a way to increase their profits by ordering more. Contrary, the truth-inducing allocations let the supplier obtain the truthful demand. However it may not be the profit-maximizing. The orders, the allocations and the prices become stable for the retailers. Overall, choosing an allocation mechanism is not just choosing one policy for allocation, but also it influences the market behaviors through a supply chain. We believe that our model represents typical phenomena in many supply chains.

This paper integrates techniques of equilibrium analysis in Economics [16, 10] for a comprehension of market rules and mechanism design of capacity allocation [6, 20, 11].

Our market model in the downstream market is similar to Kreps and Scheinkman's model [16] where sellers are in a price competition in a duopoly with pre-commitment of supply limit for each seller. The main difference is that our model has an allocation process. Once allocation is executed in our model, by treating allocated quantity as supply limit, the market behavior can be explained similarly. However,

in our model, each retailer is not able to determine allocation quantity (supply limit) which is dependent on allocation mechanisms and the market behaviors in the upstream market. Meanwhile, each seller is able to determine its supply limit independently in the Kreps and Scheinkman's model. Therefore, our model deals with more complex business scenario and it is very general transactions in daily business. Since Kreps and Scheinkman [16], several works on price competition with capacity constraints have been made. A first stream of research extends the results of the Kreps and Scheinkman. Vives [29] shows price equilibria in a symmetric oligopoly case with common capacity constraints among sellers. Francesco [10] extends the Kreps and Scheinkman model from a duopoly to an oligopoly. A second stream of research shows the limits of the Kreps and Scheinkman model by assuming asymmetric cases, including imperfect capacity pre-commitment [4, 5] and uncertain demand [25]. The difference of the two streams is caused by the symmetric behavior in the models. Our model is relevant to both streams of these literatures, since feasible allocations cover both symmetric and asymmetric cases.

The two-connected market model is common in industrial organization. The major works are vertical integration and multilateral vertical contracting in [14, 21, 24]. The aims of these papers are closely related to ours. They show how market rules in the downstream market affects the strategic choice in the upstream market and how strategic choice influences the market behavior in the downstream market. However, they do not consider capacity allocation mechanisms in the two-connected market model.

Understanding the market behaviors based on the market rule in the successive market is important to design allocation mechanisms in supply chain management, since allocation mechanisms affect market behaviors in the successive markets and the market behaviors based on the market rules affect some properties of mechanisms. This point is a main difference from some works of mechanism design for supply chain models [6, 2, 20, 30, 3] except for [11]. Therefore, our results are useful in order to design and analyze B2B marketplaces and supply chain management.

For future work, it is important to consider market mechanism design in dynamic environments. Recently, some researches propose approaches on mechanism design in a dynamic environment [8, 26, 22, 13]. However, they are restricted to a single market. We would like to propose and develop an adaptive supply chain solution on e-marketplaces for the future work.

7. REFERENCES

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